Komplexa tal

rekt. form

$a+ib \; (x+iy)$

polär form

(r, φ)

okt 27-08:01

Polär form med exponentialfunktion:

$$re^{i\varphi}$$

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Varför?

$$(x_1+iy_1)(x_2+iy_2)$$

$$= x_1x_2 - y_1y_2$$

$$+ (x_1y_2 + x_2y_1)i$$

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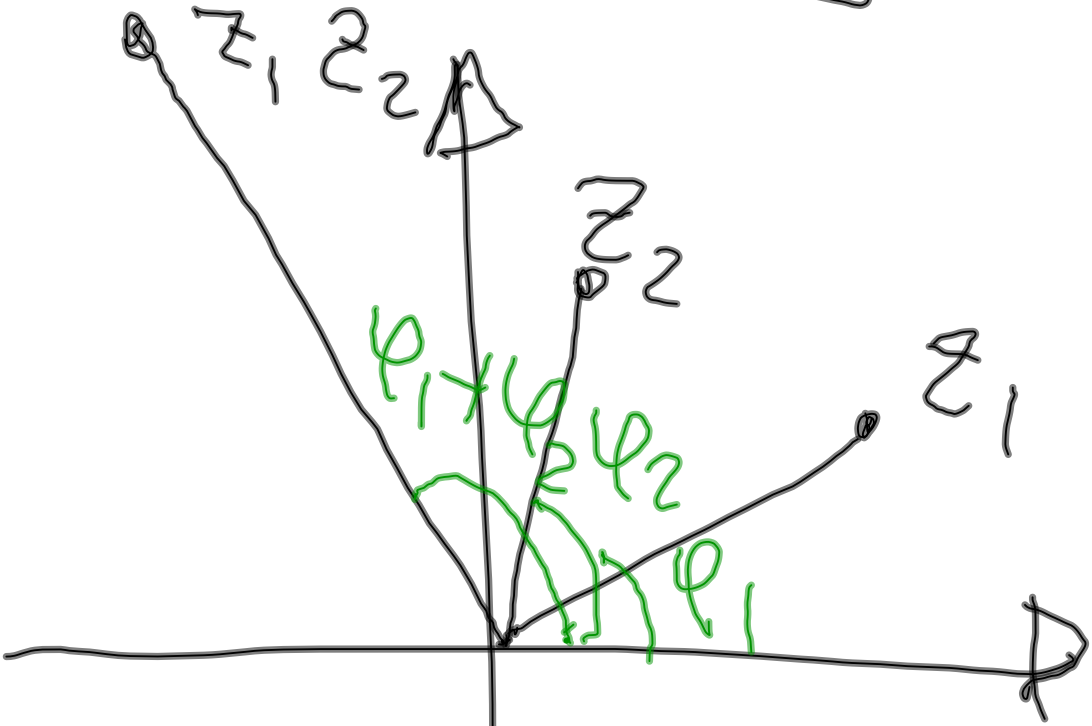
Geometriskt



$$r = |z_1 z_2| = r_1 r_2$$

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Multiplikation med polär form

$$(r_1, \varphi_1) \cdot (r_2, \varphi_2)$$

$$= (r_1 r_2, \varphi_1 + \varphi_2)$$

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exp-lag

$$e^a \cdot e^b = e^{a+b}$$

$$\left(\ln(xy) = \ln y + \ln x\right)$$

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Vi löser detta genom att skriva

$$(r_1 e^{i\varphi_1}) \cdot (r_2 e^{i\varphi_2})$$

$$= r_1 r_2 e^{i(\varphi_1+\varphi_2)}$$

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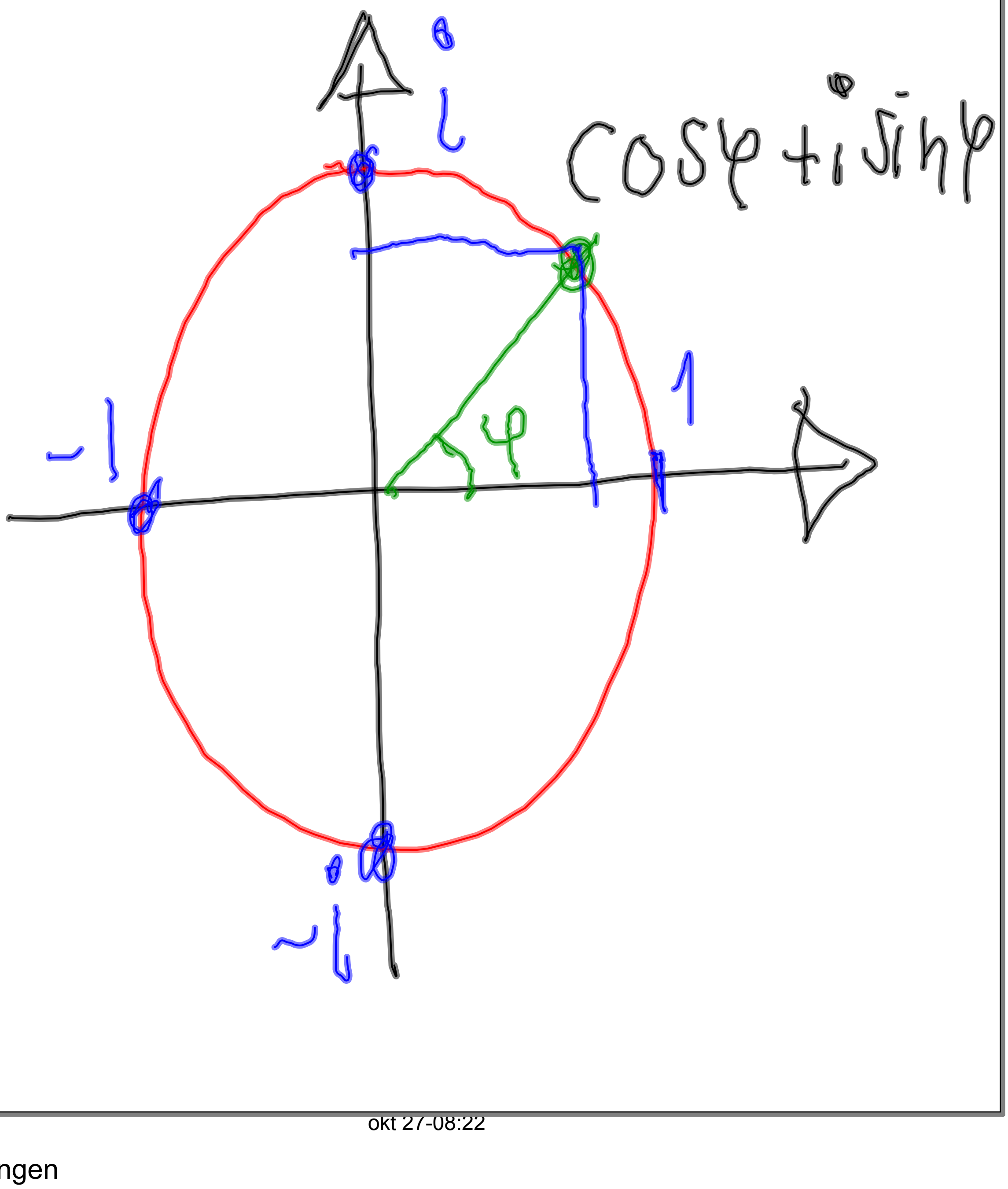
Varför blev det så?



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Multiplicera

$$(\cos\varphi + i\sin\varphi)\cdot(\cos\omega + i\sin\omega)$$

$$= \cos\varphi\cos\omega - \sin\varphi\sin\omega + i(\cos\varphi\sin\omega + \sin\varphi\cos\omega)$$

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MaD

additionslagar

$\sin(p+q) =$

$\sin p \cos q + \sin q \cos p$

$\cos(p+q) =$

$\cos p \cos q - \sin p \sin q$

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Alltså?

$$(\cos\varphi + i\sin\varphi)\cdot(\cos\omega + i\sin\omega)$$

$$= \cos(\varphi+\omega) + i\sin(\varphi+\omega)$$

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Slutsats

argumenten adderas vid multiplikation.

(argument $\leftrightarrow \varphi$)

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beloppen
multipliceras
vid multiplikation

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Test

$$z_1 = 1 + i$$

$$z_2 = 5 + i5\sqrt{3}$$

Belopp: (Pyth.)

$$|z_1| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|z_2| = \sqrt{5^2 + (5\sqrt{3})^2} = 10$$

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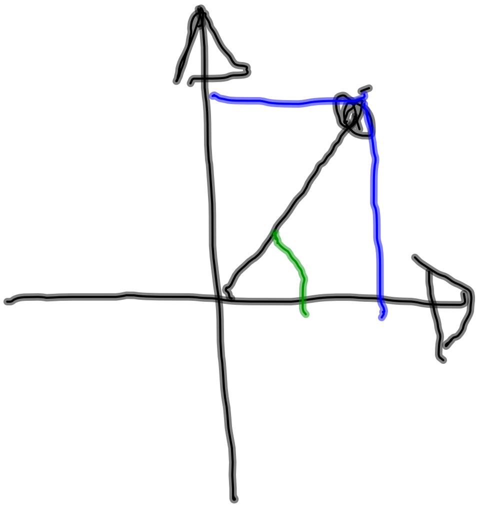
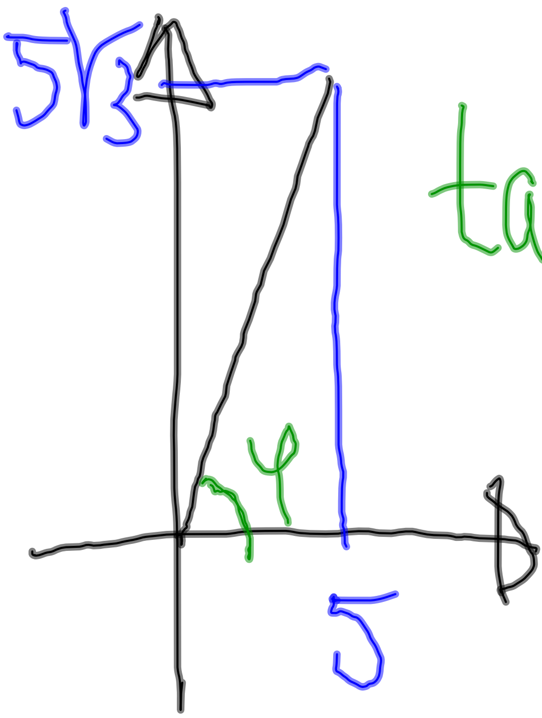
Argument



$$\arg z_1 = 45^\circ = \frac{\pi}{4}\text{ rad}$$



$$\tan\varphi = \frac{5\sqrt{3}}{5} = \sqrt{3}$$

$$\varphi = 60^\circ = \frac{\pi}{3}\text{ rad}$$

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multiplicera

$$(1+i)(5+5i\sqrt{3}) =$$

$$= 1\cdot 5 + 1\cdot 5i\sqrt{3} +$$

$$i\cdot 5 + i\cdot 5i\sqrt{3} =$$

$$= 1\cdot 5 - 5\sqrt{3}$$

$$+ i(5+5\sqrt{3})$$

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Multiplicera i polär form

$$\sqrt{2}e^{i\pi/4} \cdot 10e^{i\pi/3}$$

$$= 10\sqrt{2}e^{i(\pi/4+\pi/3)}$$

$$= 10\sqrt{2}e^{i7\pi/12}$$

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<u>Belopp $10\sqrt{2}$</u>

$$\sqrt{(5-5\sqrt{3})^2+(5+5\sqrt{3})^2}$$

$$=\sqrt{5^2+5^2+75+75}=$$

$$=\sqrt{200}=10\sqrt{2}$$

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Argumentet $7\pi/12$

$$\tan\varphi = \frac{5+5\sqrt{3}}{5-5\sqrt{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = \frac{1+2\sqrt{3}+3}{1-3} = -2-\sqrt{3}$$

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$$x^3+3x+1=$$

$$x\cdot(x^2+3)+1$$

$$=x\cdot(x\cdot x+3)+1$$

$$=1+3x+x^3=$$

$$=1+x^3+3x$$

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Ex:

$$(x-1)(x-2) =$$

$$= x \cdot x - 1 \cdot x + (-1)(-2)$$

$$+ x \cdot (-2) =$$

$$= x^2 - x + 2 - 2x$$

$$= x^2 - 3x + 2$$

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Grad

$$\operatorname{grad}(f(x)g(x))$$

$$= \operatorname{grad}(f(x)) + \operatorname{grad}(g(x))$$

om $f(x), g(x) \neq 0$

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Polynomdivision

$$f(x) = x^2 + 3x + 1$$

$$g(x) = x + 1$$

$$x\,g(x) = x^2 + x$$

$$f(x) - x\,g(x) =$$

$$= (x^2+3x\;1) - (x^2+x)$$

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$$= 3x+1-x = 2x+1$$

$$2x+1-(x+1) =$$

$$= x$$

$$2x+1-2(x+1)$$

$$= -1$$

Klar!

$\mathrm{grad}(-1) = 0$

$< \mathrm{grad}(x+1)$

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$$x^2+3x+1-x(x+1)$$

$$-2(x+1)=$$

$$=x^2+3x+1-(x+2)(x+1)$$

$$=-1$$

Alltså: $f(x)=(x+2)g(x)+(-1)$

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$$\begin{array}{r}
x+2 \\ \hline
x+1 \;\big)\; x^2+3x+1 \\
-\;\;(x^2+x) \\ \hline
2x+1 \\
-\;\;(2x+2) \\ \hline
-1
\end{array}$$

kvoten

resten

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Varför funkar faktorsatsen?

Dela $f(x)$ med $x-\alpha$ $\Rightarrow$

$$f(x) = q(x)(x-\alpha) + f(\alpha)$$

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$$f(x) = x^2 + 3x + 1$$

$$f(-1) = (-1)^2 + 3(-1) + 1$$

$$= 1 - 3 + 1 = -1$$

restem

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